CBSE

Mathematics

XIth

Trigonometric Functions

(Exercise 3.3 – Solutions)

In earlier classes, we have discussed the values of trigonometric ratios for 0°, 30°, 45°, 60° and 90°. The values of trigonometric functions for these angles are same as that of trigonometric ratios studied in earlier classes. Thus, we have the following table:

	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	– 1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	– 1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined	0	not defined	0

EXERCISE 3.3

Prove that:

1. $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$

2. $2\sin^2 \frac{\pi}{6} + \text{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$

3. $\cot^2 \frac{\pi}{6} + \text{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$

4. $2\sin^2 \frac{3\pi}{4} + 2\cos^2 \frac{\pi}{4} + 2\sec^2 \frac{\pi}{3} = 10$

5. Find the value of:

(i) $\sin 75°$ (ii) $\tan 15°$

Prove the following:

6. $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$

7. $\dfrac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\dfrac{1 + \tan x}{1 - \tan x}\right)^2$

8. $\dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$

9. $\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$

10. $\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x \cos(n + 2)x = \cos x$

11. $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}\sin x$

12. $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

15. $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

16. $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

17. $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

18. $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan\dfrac{x - y}{2}$

19. $\dfrac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

20. $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

21. $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

22. $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

23. $\tan 4x = \dfrac{4\tan x\,(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

24. $\cos 4x = 1 - 8\sin^2 x \cos^2 x$

25. $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

Q2. Q/ Prove that:

$$2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$$

Solution:

$\pi = 180^\circ$

$\frac{\pi}{3} = 60^\circ$

$\frac{\pi}{6} = 30^\circ$

We know,

$$\sin\frac{\pi}{6} = \frac{1}{2}\ ; \quad \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\operatorname{cosec} x = \frac{1}{\sin x}$$

so,

$$\operatorname{cosec}\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}}$$

$$\boxed{\sin(x+y) = \sin x\cos y + \cos x\sin y}$$

so, $\sin\left(\pi+\frac{\pi}{6}\right) = \sin\pi\cos\frac{\pi}{6} + \cos\pi\sin\frac{\pi}{6}$

$$= \left(0\times\frac{\sqrt{3}}{2}\right) + \left(-1\times\frac{1}{2}\right)$$

$$= \frac{-1}{2}$$

therefore, $\operatorname{cosec}\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}} = \frac{1}{-1/2} = -2$

- $\operatorname{cosec}^2\frac{7\pi}{6} = (-2)^2 = 4$

• $\cos\frac{\pi}{3} = \frac{1}{2}$

then,

$\Rightarrow 2\left(\frac{1}{2}\right)^2 + \cancel{4} \times \frac{1}{\cancel{4}}$

$\Rightarrow \frac{2}{4} + 1$

$\Rightarrow \frac{2+4}{4} = \frac{6}{4} = \frac{3}{2}$

LHS = RHS (Proven).

Exercise [3.3]

Q/3. Prove that :

$$\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$$

Solution:

$$\boxed{\cot x = \frac{1}{\tan x}}$$

$$\cot\frac{\pi}{6} = \frac{1}{\tan\frac{\pi}{6}} = \frac{1}{\left(\frac{1}{\sqrt{3}}\right)} = \sqrt{3}$$

- $\cot^2\cdot\frac{\pi}{6} = (\sqrt{3})^2 = 3$

- $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$$3\times\left(\frac{1}{\sqrt{3}}\right)^2 = 3\times\frac{1}{3} = 1$$

$$\boxed{\operatorname{cosec} x = \frac{1}{\sin x}}$$

- $\operatorname{cosec}\frac{5\pi}{6} = \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right)$

$$= \frac{1}{\sin\left(\pi-\frac{\pi}{6}\right)} = \frac{1}{\frac{1}{2}} = 2$$

$$\boxed{\sin(x-y) = \sin x\cos y - \cos x\sin y}$$

$$\sin\left(\pi-\frac{\pi}{6}\right) = \sin\pi\cos\frac{\pi}{6} - \cos\pi\sin\frac{\pi}{6}$$

$$= -(-1)\frac{1}{2}$$

$$= \frac{1}{2}$$

($\sin\pi = 0$
Hence first term $= 0$)

Hence,

$\Rightarrow 3 + 2 + 1 = 6$

LHS = RHS (proved). Ans.

Exercise [3.3]

Q14: Prove that

$$2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$$

Solution: $\frac{3\pi}{4} = \pi - \frac{\pi}{4}$

$\sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right)$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\sin\left(\pi - \frac{\pi}{4}\right) = \sin\pi\cos\frac{\pi}{4} - \cos\pi\sin\frac{\pi}{4}$

$\sin\pi = 0$; so first term will be $= 0$

$\sin\left(\pi - \frac{\pi}{4}\right) = -\cos\pi\sin\frac{\pi}{4}$

$= -(-1)\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

- $\sin\frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$
- $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$
- $\sec x = \frac{1}{\cos x}$

$\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

Now, putting these

$\Rightarrow 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times(2)^2$

$\Rightarrow 1 + 1 + 8 = 10$ LHS = RHS (Proven.)

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Exercise (3·3)

Q5) find Value of $\sin 75^\circ$

$\sin 75^\circ = \sin(45+30)$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \sin 45^\circ \cos 30 + \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans

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Exercise [3.3]

Q/6: Prove:

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$$

*
$$\boxed{\begin{array}{l} \text{i)}\ \cos(x+y)=\cos x\cos y-\sin x\sin y \\ \text{ii)}\ \cos x\cos y=\cos(x+y)+\sin x\sin y \end{array}}$$ (Side changed)

Here, $x=\left(\frac{\pi}{4}-x\right)$; $y=\left(\frac{\pi}{4}-y\right)$

From (ii) above

$$\Rightarrow \cos\left(\frac{\pi}{4}-x+\frac{\pi}{4}-y\right)+\cancel{\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)}$$
$$-\cancel{\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)}$$

$$\Rightarrow \cos\left(\frac{\pi}{4}+\frac{\pi}{4}-x-y\right)$$

$$\Rightarrow \cos\left(\frac{\pi}{2}-(x+y)\right) \quad \Rightarrow \cos\left(\frac{\pi}{2}-(x+y)\right)=\sin(x+y)$$

*
$$\boxed{\cos\left(\frac{\pi}{2}-x\right)=\sin x}$$

$\underline{LHS}=\underline{RHS}$

(thus Proven.)

from Here, $x\Rightarrow(x+y)$

Q7)

$$\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

Sol:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\boxed{\tan\frac{\pi}{4} = 1}$$

$$\Rightarrow \frac{\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}} = \frac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}}$$

$$\Rightarrow \frac{1+\tan x}{1-\tan x} \times \frac{1+\tan x}{1-\tan x} = \frac{(1+\tan x)^2}{(1-\tan x)^2}$$

$$\Rightarrow \left(\frac{1+\tan x}{1-\tan x}\right)^2$$

LHS = RHS (proven)

Prove:

98/ $$\frac{\cos(\pi+x)\ \cos(-x)}{\sin(\pi-x)\ \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$$

Solⁿ

- $\cos(\pi+x) = -\cos x$
- $\cos(-x) = \cos x$
- $\sin(\pi-x) = \sin x$
- $\cos\left(\frac{\pi}{2}+x\right) = -\sin x$

we know that

$$\Rightarrow \frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}$$

$$\Rightarrow \frac{-\cos^2 x}{-\sin^2 x} = \cot^2 x$$

$\Rightarrow$ LHS = RHS (proven)

XI^{th} – Maths – Trigonometric Functions

pg(72). Example (17)

Q/. Prove that

$$\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$$

Sol: L.H.S.

$$\Rightarrow \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$$

$$\Rightarrow \frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x} \quad \text{(rearrange)}$$

$$\boxed{\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}$$

$\underline{\sin 5x + \sin x}$

$(x \Rightarrow 5x \; ; \; y \Rightarrow x)$

$$\sin 5x + \sin x = 2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2}$$

$$= 2\sin\frac{6x}{2}\cos\frac{4x}{2}$$

$$= 2\sin 3x \cos 2x$$

put this value in the main equation.

$$\boxed{\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}}$$

$\underline{\cos 5x - \cos x}$

$x \Rightarrow 5x\ ;\ y \Rightarrow x$

$$\cos 5x - \cos x = -2\sin\frac{5x+x}{2}\sin\frac{5x-x}{2}$$

$$= -2\sin\frac{6x}{2}\sin\frac{4x}{2}$$

$$= -2\sin 3x \sin 2x$$

put this value in the main equation.

$$\Rightarrow \frac{2\sin 3x\cos 2x - 2\sin 3x}{-2\sin 3x\sin 2x}$$

(divide by 2)

$$\Rightarrow \frac{\sin 3x\cos 2x - \sin 3x}{-\sin 3x\sin 2x}$$

$$\Rightarrow \frac{\sin 3x(\cos 2x - 1)}{-\sin 3x\sin 2x}$$

$$\Rightarrow \frac{(\cos 2x - 1)}{-\sin 2x} = \frac{1-\cos 2x}{\sin 2x}$$

$$\boxed{\sin 2x = 2\sin x\cos x}$$

$$\boxed{\cos 2x = 1 - 2\sin^2 x}$$

OR

$$\boxed{1 - \cos 2x = 2\sin^2 x}$$

$$\Rightarrow \quad \frac{2\sin^2 x}{2\sin x\cos x}$$

$$\Rightarrow \quad \frac{\cancel{2}\sin^2 x}{\cancel{2}\sin x\cos x}$$

$$\Rightarrow \quad \frac{\sin x}{\cos x} = \tan x$$

LHS = RHS. (proven)

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Thank you!

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