MATHEMATICS

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2024

Limits of Accuracy

Upper and Lower bounds

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Mathematics - Limits of Accuracy

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Limits of Accuracy - Upper and Lower bounds

- Measurements are always given to a certain level of accuracy.
- The **limits of accuracy** of any measurement **depends** on the **instruments used** to measure it,
- But even with very accurate scientific measuring devices, quantities cannot be measured exactly because they are **continuous values**.

Limits of Accuracy - Upper and Lower bounds

- The way we **round numbers** *means* that all measurements have to be **between certain limits**.
- These limits depend on the **level of accuracy** that was used to round the measurement.

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Limits of Accuracy - Upper and Lower bounds

- The **smallest value** a measurement can take is called the lower bound.

- The **greatest value** it can take is called the upper bound.

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Limits of Accuracy - Upper and Lower bounds

- The difference between the upper and lower bounds is called the **error interval** - a measurement can fall anywhere within this interval.

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Example - How these bounds apply in real life.

Raeman operates a crane at a building site. The manufacturer states that the crane can safely lift a maximum of 6.2 tonnes. Raeman needs to move some 1.2 tonne blocks, where the mass of each block is given to 1 decimal place. He starts by calculating how many 1.2 tonne blocks make 6.2 tonnes in total.

$$\frac{6.2}{1.2} = 5.17 \text{ blocks}$$

so Raeman decides he can **safely** lift 5 blocks.

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Example - How these bounds apply in real life.

But, the mass of the blocks has been rounded to 1 decimal place. What if the blocks all weigh **1.245 tonnes**? They all round to 1.2 tonnes to 1 decimal place, but the total mass of 5 blocks is now:

$$5 \times 1.245 = 6.225 \text{ tonnes.}$$

This is more than the crane can lift safely.

How can Raeman work out the maximum number of blocks he can lift safely?

Example - Solution

Finding the greatest and least possible values of a rounded measurement

In the crane example **1.2 tonnes** is rounded to **1 decimal place**. It can be useful to work out the **greatest** and **least possible** values of the actual measurement.

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Example - Solution

Finding the greatest and least possible values of a rounded measurement

If you place 1.2 on a number line, you can see much more clearly what the range of possible values are:

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<u>Example</u> - Solution

At the **upper end**, the range of possible values stops at **1.25**. If you round 1.25 to 1 decimal place you get **1.3**. Although 1.25 does ***not*** round to **1.2** (to 1 decimal place), it is still used as the upper value.

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Example - Solution

However, you should understand that the actual value can be anything *up to but not including* **1.25**.

The **largest possible value** is called the **upper bound**. Similarly, the **lowest possible value** is called the **lower bound.**

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Example - Solution

If ***m*** presents the mass (in tonnes) of the block, then you can express the error interval as: $\mathbf{1.15 \leq m < 1.25}$

This is called inequality notation and it shows that the true value of ***m*** lies between 1.15 (including 1.15) and 1.25 (not including 1.25).

Example - Solution

So, to safely lift the blocks

If the **upper bound** is taken as **1.24**
(it will round to **1.2** - *upto 1 decimal place*)

Total weight 1.24 X **5** = 6.2 tonnes

In that case, Raeman can safely lift 5 blocks.

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IMPORTANT POINT

To find the limits think about **what 1 decimal place means** - one decimal place is **0.1**.

If you divide **0.1 by 2** you get **0.05**. **Add 0.05** to find the **upper limit** and **subtract 0.05** to find the **lower limit**, Write the boundaries as an **inequality**.

$$1.15 \leqslant \overset{+0.05}{\rule{3em}{0.4pt}} 1.2 \overset{+0.05}{\rule{3em}{0.4pt}} < 1.25$$

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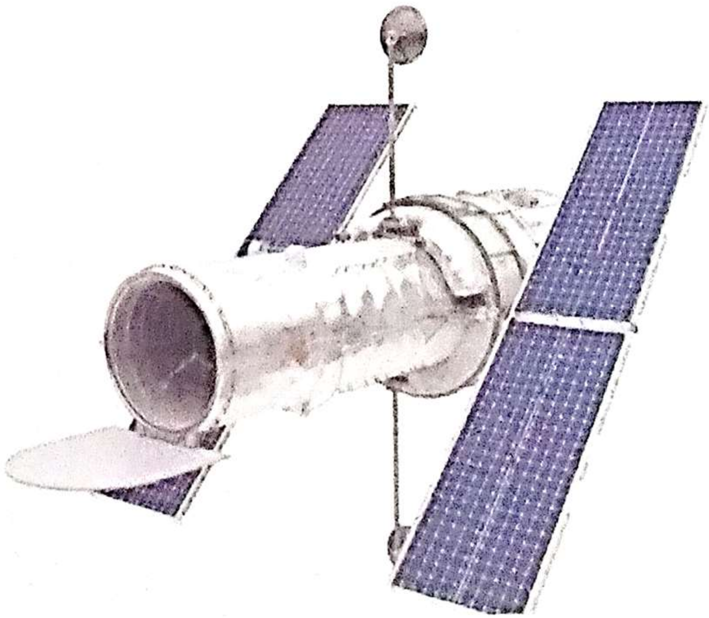
LINK

The Hubble Space Telescope, launched into space in 1990, was originally made with a 1 mm error in the shape of the lens. This caused many of the images to be blurred and a repair had to be made in 1993. Even a small error can completely ruin scientific instruments that rely on precise measurement.



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1. Questions with Solutions

Find the upper and lower bounds of the following, taking into account the level of rounding shown in each case.

a 10 cm, to the nearest cm

b 22.5, to 1 decimal place

c 128 000, to 3 significant figures

d 120, to the nearest 20

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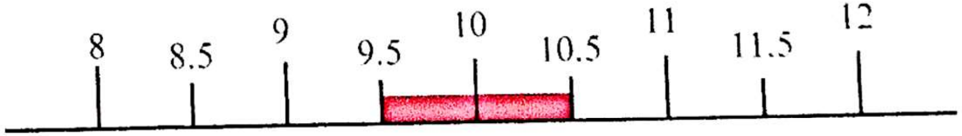
1. Questions with Solutions

Answers

a Show 10 cm on a number line.

The real value will be closest to 10 cm if it lies between the lower bound of 9.5 cm and the upper bound of 10.5 cm.



b Look at 22.5 on a number line.

The real value will be closest to 22.5 if it lies between the lower bound of 22.45 and the upper bound of 22.55.

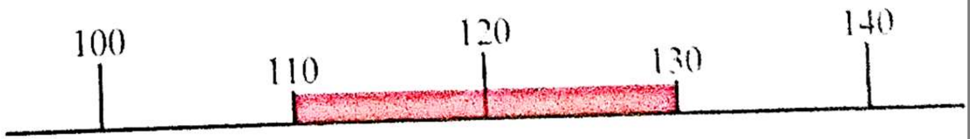
c 128 000 is shown on a number line.

128 000 lies between the lower bound of 127 500 and the upper bound of 128 500.

1. Questions with Solutions

d 120 is shown on a number line.

120 lies between the lower bound of 110 and the upper bound of 130.



2. Questions with Solutions

If $a = 3.6$ (to 1dp) and $b = 14$ (to the nearest whole number), find the upper and lower bounds for each of the following:

a $a + b$ **b** ab **c** $b - a$ **d** $\frac{a}{b}$ **e** $\frac{a+b}{a}$

Firstly, find the upper and lower bounds of a and b:

$3.55 \leqslant a < 3.65$ and $13.5 \leqslant b < 14.5$

2. Solutions

a

Upper bound for $(a+b)$ = upper bound of a + upper bound of b

$= 3.65 + 14.5$

$= 18.15$

Lower bound for $(a+b)$ = lower bound for a + lower bound for b

$= 3.55 + 13.5$

$= 17.05$

This can be written as: $17.05 \leqslant (a+b) < 18.15$

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2. Solutions

b

Upper bound for ab = upper bound for $a \times$ upper bound for b

$= 3.65 \times 14.5$

$= 52.925$

Lower bound for ab = lower bound for $a \times$ lower bound for b

$= 3.55 \times 13.5$

$= 47.925$

This can be written as: $47.925 \leqslant ab < 52.925$

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c Think carefully about $b - a$. To find the upper bound you need to subtract as small a number as possible from the largest possible number. So:

Upper bound for $(b - a)$ = upper bound for b lower bound for a

$= 14.5 - 3.55$

$= 10.95$

Similarly, for the lower bound:

Lower bound $(b - a)$ = lower bound for b upper bound for a

$= 13.5 - 3.65$

$= 9.85$

This can be written as: $9.85 \leqslant (b - a) < 10.95$

2. Solutions

d

To find the upper bound of $\frac{a}{b}$ you need to divide the largest possible value of a by the smallest possible value of b:

$$\text{Upper bound} = \frac{\text{upper bound for } a}{\text{lower bound for } b} = \frac{3.65}{13.5} = 0.2703\ldots = 0.270 \text{ (3sf)}$$

$$\text{Lower bound} = \frac{\text{lower bound for } a}{\text{upper bound for } b} = \frac{3.55}{14.5} = 0.2448\ldots = 0.245 \text{ (3sf)}$$

This can be written as: $0.245 \leqslant \frac{a}{b} < 0.270$

2. Solutions

e

$$\text{Upper bound of } = \frac{a+b}{a} = \frac{\text{upper bound of } a+b}{\text{lower bound of } a} = \frac{18.15}{3.55} = 5.1126\ldots = 5.11 \text{ (3sf)}$$

$$\text{Lower bound of } = \frac{a+b}{a} = \frac{\text{lower bound of } a+b}{\text{upper bound of } a} = \frac{17.05}{3.65} = 4.6712\ldots = 4.67 \text{ (3sf)}$$

This can be written as: $4.67 \leqslant \dfrac{a+b}{a} < 5.11$

Thank you!

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