INTRODUCTION TO PROBABILITY

• Event • Probability • Probability scale • Trial • Experimental probability • Theoretical probability • Outcome • Favourable outcomes • Bias • Possibility diagram • Mutually exclusive • Independent

▪ PROBABILITY IN EVERYDAY LIFE

- What is the chance that it will rain tomorrow?
- When you flip a coin to decide which team will start a match, how likely is it that you will get a head?

Questions of chance come into our everyday life from - what is the weather going to be like tomorrow to - who is going to wash the dishes tonight. Words like ‘certain’, ‘even’ or ‘unlikely’ are often used to roughly describe the chance of an event happening but **probability** refines this to numbers to help make more accurate predictions.

Calculating probability

Probability always has a value between 0 and 1.

The sum of probabilities of mutually exclusive events is 1.

$$\text{Probability} = \frac{\text{number of successful outcome}}{\text{total number of outcomes}}$$

Relative frequency

The number of times an event occurs in a number of trials is its relative frequency.

Relative frequency is also called experimental probability.

$$\text{Relative frequency} = \frac{\text{number of times an outcome occurred}}{\text{number of trials}}$$

▪ BASIC PROBABILITY

Events

When you roll a die, you may be interested in throwing a prime number. When you draw a name out of a hat, you may want to draw a boy’s name. Th rowing a prime number or drawing a boy’s name are examples of ***events***.

Probability always has a value between 0 and 1

Probability is a measure of how likely an event is to happen. Something that is **impossible** has a value of **zero** and something that is **certain** has a value of **one**. The range of values from **zero to one** is called a **probability scale**. A probability cannot be negative or be greater than one.

[0 – 1]
The smaller the probability, the closer it is to zero and the less likely the associated event is to happen. Similarly, the higher the probability, the more likely the event.

A die is the singular of dice.

▪ **Trail and Experimental Probability**
Performing an experiment, such as rolling a die, is called a **trial**. If you repeat an experiment, by carrying out *a number of trials*, then you can find an **experimental probability** of an event happening: this fraction is often called the **relative frequency**.

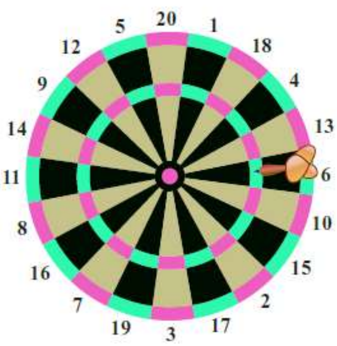
$$P(A) = \frac{\text{number of times desired event happens}}{\text{number of trials}}$$

or, sometimes:

$$P(A) = \frac{\text{number of successes}}{\text{number of trials}}$$

P(A) means the probability of event A happening.

Usually ‘AND’ in probability means you will need to multiply probabilities. ‘OR’ usually means you will need to add them.



Example1

Suppose that a blindfolded man is asked to throw a dart at a dartboard.
If he hits the number six 15 times out of 125 throws, what is the probability of him hitting a six on his next throw?

$$P(\text{six}) = \frac{\text{number times a six obtained}}{\text{number of trials}} = \frac{15}{125} = 0.12$$

▪ RELATIVE FREQUENCY and EXPECTED OCCURRENCES

You can use relative frequency to make ***predictions*** about what might happen in the future or how often an event might occur in a larger sample. For example, if you know that the relative frequency of rolling a **4** on particular die is 18%, you can work out how many times you'd expect to get **4** when you roll the dice 80 or 200 times.

18% of 80 = 14.4 and 18% of 200 = 36, so if you rolled the same die 80 times you could expect to get a 4 about 14 times and if you rolled it 200 times, you could expect to get a **4** thirty-six times (36 times).

Remember though, that even if you expected to get a **4** thirty-six times (36 times), this is not a given and your *actual results may be very different*.

▪ THEORETICAL PROBABILITY / EXPECTED PROBABILITY

When you flip a **coin** you may be interested in the event 'obtaining a head' but this is only one possibility. When you flip a coin there are two possible outcomes: 'obtaining a head' or 'obtaining a tail.'

You can calculate the theoretical (or expected) probability easily if all of the possible outcomes are ***equally likely***, by counting the number of favourable outcomes and dividing by the number of possible outcomes. **Favourable outcomes** are any outcomes that mean your event has happened.

For example, if you throw an **unbiased die** and need the probability of an **even number**, then the favourable outcomes are **two**, **four** or **six** (2,4,6). There are three of them.
Under these circumstances the event A (obtaining an even number) has the probability:

$$P(A) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}} = \frac{3}{6} = \frac{1}{2}$$

Never assume that a die or any other object is unbiased unless you are told that this is so.

Biased

Of course! a die may be weighted in some way, or imperfectly made, and indeed this may be true of any object discussed in a probability question. Under these circumstances a die, coin or other object is said to be ***biased***. The outcomes will no longer be equally likely and you may need to use experimental probability.

Biology students will sometimes consider how genes are passed from a parent to a child. There is never a certain outcome, which is why we are all different. Probability plays an important part in determining how likely or unlikely a particular genetic outcome might be.

Example2

An unbiased die is thrown and the number on the upward face is recorded. Find the probability of obtaining:

a a three **b** an even number **c** a prime number.

a	$P(3) = \frac{1}{6}$	There is only one way of throwing a three, but six possible outcomes (you could roll a 1, 2, 3, 4, 5, 6).
b	$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$	There are three even numbers on a die, giving three favourable outcomes.
c	$P(\text{prime number}) = \frac{3}{6} = \frac{1}{2}$	The prime numbers on a die are 2, 3 and 5, giving three favourable outcomes.

Example3

A card is drawn from an ordinary 52 card pack. What is the probability that the card will be a king?

$P(\text{King}) = \frac{4}{52} = \frac{1}{13}$	Number of possible outcomes is 52. Number of favourable outcomes is four, because there are four kings per pack.

Example4

Jason has 20 socks in a drawer.

8 socks are red, 10 socks are blue and 2 socks are green. If a sock is drawn at random, what is the probability that it is green?

$P(\text{green}) = \frac{2}{20} = \frac{1}{10}$	Number of possible outcomes is 20. Number of favourable outcomes is two.

Example5

Nine painters are assigned a letter from the word HOLLYWOOD for painting at random. Find the probability that a painter is assigned:

a the letter 'Y' **b** the letter 'O' **c** the letter 'H' or the letter 'L' **d** the letter 'Z'.

For each of these the number of possible outcomes is 9.

a	$P(Y) = \frac{1}{9}$	Number of favourable outcomes is one (there is only one 'Y').
b	$P(O) = \frac{3}{9} = \frac{1}{3}$	Number of favourable outcomes is three.
c	$P(H \text{ or } L) = \frac{3}{9} = \frac{1}{3}$	Number of favourable outcomes = number of letters that are *either* H or L = 3, since there is one H and two L's in Hollywood.
d	$P(Z) = \frac{0}{9} = 0$	Number of favourable outcomes is zero (there are no 'Z's)

▪ THE PROBABILITY THAT AN EVENT DOES NOT HAPPEN

Something ***may happen*** or it ***may not happen***. The probability of an *event happening* may be different from the probability of the *event not happening* but the two combined probabilities will ***always sum up to one***.

If A is an event, then $\bar{A}$ is the event that A does ***not*** happen and **P($\bar{A}$) = 1− P(A)**

$\bar{A}$ is usually just pronounced as 'not A'.

Example6

The probability that Jasmine passes her driving test is $\frac{2}{3}$. What is the probability that Jasmine fails?

$P(\text{failure}) = 1 - \frac{2}{3} = \frac{1}{3}$	P(failure) = P(not passing) = 1 – P(passing)

▪ INDEPENDENT AND MUTUALLY EXCLUSIVE EVENTS

Independent Events

If you flip a **coin** once the probability of it showing a head is 0.5. If you flip the coin a *second time* the probability of it showing a head is still 0.5, regardless of what happened on the first flip. Events like this, where the first outcome has no influence on the next outcome, are called independent events.

Sometimes there can be more than one stage in a problem and you may be interested in what combinations of outcomes there are. If A and B are independent events then:

P(A happens and then B happens) = P(A) × P(B)
or
P(A and B) = P(A) × P(B)

#Note that this formula is only true if A and B are independent.

Mutually Exclusive Events

There are situations where it is impossible for events A and B to happen at the same time. For example, if you throw a normal die and let:

A = the event that you get an *even number*
and
B = the event that you get an *odd number*

then *A and B cannot happen together* because no number is both even and odd *at the same time*. Under these circumstances you say that A and B are **mutually exclusive** events and **P(A or B) = P(A) + P(B)**.
#Note, this formula only works if A and B are mutually exclusive.

The following worked examples demonstrate how these simple formulae can be used.

Example7

Simone and Jon are playing darts. The probability that Simone hits a bull's-eye is 0.1. The probability that Jon throws a bull's-eye is 0.2. Simone and Jon throw one dart each. Find the probability that:

a both hit a bull's-eye **b** Simone hits a bull's-eye but Jon does not
c exactly one bull's-eye is hit.

Simone's success or failure at hitting the bull's-eye is independent of Jon's and vice versa.

a P(both throw a bull's-eye) $= 0.1 \times 0.2 = 0.02$

b P(Simone throws a bull's-eye but Jon does not) $= 0.1 \times (1 - 0.2) = 0.1 \times 0.8 = 0.08$

c P(exactly one bull's-eye is thrown)

= P(Simone throws a bull's-eye and Jon does not or Simone does not throw a bull's-eye and Jon does)

$= 0.1 \times 0.8 + 0.9 \times 0.2$

$= 0.08 + 0.18$

$= 0.26$

Example8

James and Sarah are both taking a music examination independently. The probability that James passes is $\frac{3}{4}$ and the probability that Sarah passes is $\frac{5}{6}$.

What is the probability that:

a both pass **b** neither passes **c** at least one passes
d either James or Sarah passes (not both)?

Use the formula for combined events in each case.

Sarah's success or failure in the exam is independent of James' outcome and vice versa.

a P(both pass) = P(James passes and Sarah passes) $= \frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8}$

b P(neither passes) = P(James fails and Sarah fails)

$$= (1 - \frac{3}{4}) \times (1 - \frac{5}{6})$$

$$= \frac{1}{4} \times \frac{1}{6}$$

$$= \frac{1}{24}$$

c P(at least one passes) = 1 − P(neither passes) $= 1 - \frac{1}{24} = \frac{23}{24}$

d P(either Sarah or James passes)

= P(James passes and Sarah fails or James fails and Sarah passes)

$$= \frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}$$

$$= \frac{3}{24} + \frac{5}{24}$$

$$= \frac{8}{24}$$

$$= \frac{1}{3}$$

The events, 'James passes and Sarah fails' and, 'James fails and Sarah passes,' are mutually exclusive because no-one can both pass and fail at the same time. This is why you can add the two probabilities here.

▪ WHAT WE HAVE LEARNED … [SUMMARY]

- Probability measures how likely something is to happen.
- An outcome is the single result of an experiment.
- An event is a collection of favourable outcomes.
- Experimental probability can be calculated by dividing the number of favourable outcomes by the number of trials.
- Favourable outcomes are any outcomes that mean your event has happened.
- If outcomes are equally likely then theoretical probability can be calculated by dividing the number of favourable outcomes by the number of possible outcomes.
- The probability of an event happening and the probability of that event not happening will always sum up to one.
- If A is an event, then $\bar{\mathbf{A}}$ is the event that A does ***not*** happen and P($\bar{\mathbf{A}}$) = 1− P(A)
- Independent events do not affect one another.
- Mutually exclusive events cannot happen together.